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A Structural Model of an Exemplary Mathematics Teacher in Junior High School

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Abstract. This study develops a structural model of an exemplary mathematics teacher featuring students' perspectives. This study used an exploratory sequential mixed method research (MMR) design. Focus Group Discussions (FGDs) and Key Informant Interviews (KIIs) were conducted to determine the attributes of an exemplary mathematics teacher. The participants of the FGD were the students from the chosen two secondary schools, with eight student representatives from each school. The KII was also done with ten mathematics teachers and one mathematics supervisor. Exploratory Factor Analysis (EFA) was conducted for the attributes of an exemplary mathematics teacher, and it yielded four factors, namely: (1) Expertise, (2) Effective Communication with Students, (3) Devolution, and (4) Teacher's Disposition. Confirmatory Factor Analysis (CFA) was performed on the hypothesized Model with four latent variables with their corresponding observed variables using Analysis of Moment Structure (AMOS) version 21. Using the Structural Equation Model (SEM) technique, the findings revealed a significant influence of the factors on exemplary mathematics teaching, thus providing a theoretical foundation for this study. Moreover, the findings also showed significant contributions of the observed variables (S.Q. items) to the latent variables (factors). Based on the SEM findings supported by the literature, the structural Model of an exemplary mathematics teacher was empirically founded. The developed model could serve as one of the platforms for lifting mathematics education. Also, the validated survey questionnaire could be a tool for assessing mathematics teachers.

Keywords: Confirmatory factor analysis; Exemplary Mathematics teaching; Structural equation model; Students' perspectives.

1.0 Introduction

Numerous studies have explored the integrity of mathematics teaching, but very few have sought the perspective of learners. Students' perspectives on mathematics teaching are deemed significant, given the current international trends of declining engagement and participation of students in this subject area. This academic matter has always been regarded as the primary concern of nations struggling to increase mathematics performance. Most of the studies that gathered students' responses about disengagement from mathematics have reported low confidence in the subject, poor content knowledge, lack of motivation in learning, and lastly, dissatisfaction with the skills and knowledge of the teacher (Aguilar, 2021).

In trying to look at situations worldwide, the quality of teaching makes up for significant Variance in students' mathematics performance (Toropova et al., 2019). According to Zhou et al. (2023), students in China who have exemplary teachers perform better than those whose inefficient teachers handle. Many countries such as Australia, the Netherlands, France, Finland, and the United States, which previously focused on just hiring the required number of teachers, are now inclined to ensure that quality of teaching should be the essential thing to consider in the educational policy (OECD, 2015).

Moreover, the quality of mathematics teaching has a concrete reflection on students' mathematics performance. Program for International Student Assessment [PISA] (2022) reported that Singapore was recognized as the topmost performing country in mathematics assessment while the Philippines is still one of the countries with low performance in mathematics, reading, and science. The Philippines took part in PISA 2018 and 2022 conducted by OECD. Filipino students are among the least proficient in arithmetic, reading, and science. Its performance has not considerably improved since the review in 2018 (Acido & Caballes, 2024). This is confirmed by the study of Kilag et al. (2024) that fundamental issues in mathematics education have not been sufficiently addressed. Filipino pupils' average PISA results have improved slightly, rising from 353 in 2018 to 355 in 2022. Despite efforts to improve, the marginal result still placed the Philippines among the lowest performers, ranking 75th out of 81 countries (OECD, 2023). This highlights a substantial gap in essential mathematics skills and critical thinking ability, with nearly 84% of students still below the basic competency level.

This inefficient performance in mathematics has been an alarming concern in recent years. Several factors contribute to the poor performance of Filipino students in mathematics. One concern is the quality of mathematics instruction. Research revealed that teachers' expertise in teaching mathematics is essential for filling in the hole in mathematics performance (Katz & Stupel, 2016). In the Philippines, however, many teachers lack professional development training in teaching lifelong mathematics skills (Tan, 2015). Roxas (2015) emphasized that the most significant factor in creating interest in mathematics is a systematic, well-rounded, competent, and expert teacher. This mathematics teaching gap requires a mathematics teacher's full potential and expertise.

In line with the idea of uplifting students' performance in mathematics to national and international standards, the need for an exemplary mathematics teacher should be given utmost attention. Several studies posited a definition of an exemplary teacher. The terms 'expert teachers', 'excellent teachers,' and 'lead teachers' were utilized to describe exemplary teachers and academic leaders who exemplify the establishment of a meaningful teaching-learning process in the classroom. Webb et al. (2019) deal with the idea that exemplary teachers are not only capable of teaching and facilitating students but are also able to collaborate with colleagues in order to widen their perspective on matters regarding instruction. Moreover, these teachers are considered fully equipped in their field of specialization (Bush et al., 2016). The main task of an exemplary mathematics teacher is to ensure that they deliver instruction with high standards.

Moreover, the expert teacher vouches for all components of excellent mathematics teaching, such as the possession of highly equipped content knowledge, mastery of content, mathematics instructional techniques, and overall teaching strategy so that the teacher can effectively interact with the students (Science Education Institute, Department of Science and Technology [SEI-DOST] and the Philippine Council of Mathematics Teacher Education [MATHTED] (2011). This type of teacher supremely dissolves theory in actual teaching. The exemplary teacher also manifests the dire need to transcend beyond the goals of mathematics education and is willing to continue the development of mathematics teaching and learning.

In line with advocating excellent mathematics teaching, the Department of Education recognizes teachers who go the extra mile to deliver quality mathematics education at par with other nations. Also, giving recognition to exemplary teachers and granting salary and vertical promotion are deemed necessary to increase their motivation to achieve local, national, and even international standards of performance among students (Ariffin et al., 2018). This is also confirmed by Symaco (2013) that ensuring adequate funding, providing incentives for teacher performance, and establishing closed monitoring and evaluation frameworks are essential measures in uplifting the mathematics proficiency of students.

However, minimal research exists on the roles and responsibilities of Excellent Mathematics Teachers that could contribute to developing a performance framework specifically for teaching mathematics, wherein students' perceptions will be valued and considered. Hence, this study aimed to develop a framework for Exemplary Mathematics Teachers' roles and responsibilities in response to the need to enhance the mathematics proficiency level holistically and, at the same time, construct and validate a students' evaluation tool for assessing their mathematics teachers' performance.

This study aimed to develop a structural model of an exemplary Mathematics teacher at the junior high school level. The findings of this study would serve as a foundation for developing an annual program to train mathematics teachers inclined to provide efficient teaching strategies. This would also help monitor the performance of mathematics teachers. The findings of this study would also encourage math teachers to continue filling in the gap between students and mathematics learning and eradicating the notion of difficulty of the subject in order to direct them to the required proficiency level in mathematics.

2.0 Methodology

2.1 Research Design

The study used an exploratory sequential mixed method research (MMR) design. The exploratory design of research involved qualitative data. The resulting themes were used to fabricate a quantitative instrument to further explore the research objectives (Creswell & Plano Clark, 2011; Teddlie & Tashakkori, 2008; Onwuegbuzie et al., 2010). The qualitative method was utilized to generate the attributes of an exemplary mathematics teacher through focus group discussions (FGDs) and Key Informant Interviews (KIIs). The output of the FGDs and KIIs was used to construct the framework of the study, which would also be the basis for the survey instrument.

2.2 Research Locale

This study was conducted in Davao del Sur. Nine municipalities existed in Davao del Sur, each with one private high school and one public high school. The sectarian private schools were managed by different Catholic religious organizations, while the public schools were managed by the Department of Education.

2.3 Research Participants

For the qualitative aspect of the study, focus group discussions (FGDs) and key informant interviews (KIIs) were conducted. The participants of the FGD were the students from the chosen two secondary schools in Davao del Sur, one private school and one public school with the highest population of students. Each school had eight representatives with equal participation of students based on their percentile rank in mathematics grades. The students with the highest rank, the middle, and the bottom were taken as participants in the FGD. This was to ensure that the three levels of performance were well-represented. The KII was also done with ten mathematics teachers and one mathematics supervisor. This was to enrich the gathering of attributes of an exemplary mathematics teacher. KIIs are qualitative. These were in-depth interviews with people who were knowledgeable about the subject. The purpose of conducting KII was to gather information from participants with different backgrounds to deepen the discussion and, in turn, ask follow-up questions.

A purposive sampling method was used to determine all the FGD members and key informants. For FGD, students at the top (three students), middle (three students), and bottom (two students) percentile ranks based on mathematics grades were the participants. Regarding the recommended sample size for the qualitative research, Subedi (2021) said that four to five participants are enough to have enough knowledge and expertise in the topic of inquiry. Mathematics teachers from two chosen schools for FGDs were randomly chosen as key informants so that the needed information to expedite the study would be the exact data necessary to answer the research questions. One mathematics supervisor was also interviewed to widen the data sources scope further.

For the quantitative aspect of the study, the respondents were Grades 7-10 junior high school students. The respondents were 416 students from private and public high schools in Davao del Sur. The results of the FGD served as the basis for crafting the survey questionnaire, which was validated and pilot-tested before its full implementation.

To select the student respondents, the researcher utilized an online-based sample size calculator to determine the sample size. Then, proportionate stratified random sampling was used to determine the specific number of respondents who would answer the questionnaires. This would ensure a fair selection of students.

2.4 Research Instrument

The researcher developed an FGD template and a KII semi-structured interview form. These instruments helped the researcher gather the participants' responses to questions about their perceptions of the attributes of an exemplary mathematics teacher. After gathering these data, a survey questionnaire was developed. There were 91 formulated meanings constructed from the responses of the participants in FGDs and KIIs. Thus, these 91 meanings became the items for the survey questionnaire. Three mathematics experts validated the instrument. After receiving experts' feedback, the researcher revised the questionnaire following their comments and suggestions to merge similar items; hence, the number of items was trimmed down to 65. The questionnaire was then administered to 42 junior high school students for pilot testing. The purpose of testing was to detect any problems with the items. A Cronbach's Alpha 0.943 made the survey questionnaire ready for implementation.

The researcher utilized a 7-point Likert Scale in the survey questionnaire. Students rated each item, one being the lowest and seven being the highest. Tarka (2017) suggested that a 7-point scale is comparable to analytical tools such as confirmatory factor analysis or structural equation models. Table 1 shows the selected examples from the survey questionnaire. Four items were selected as examples: 24, 62, 16, and 18. The first example pertains to the pedagogical knowledge a math teacher should have. The second one points to a teacher's motivating students to show their solutions in any method they know. The following example pertains to applying student-centered learning. The last example concerns the degree of patience a mathematics teacher should have.

Table 1. Example of items in the survey questionnaire

Item Number	Statements
24	*An exemplary mathematics teacher knows when to apply a particular strategy best and continues to search for more effective teaching techniques/strategies.*
62	*An exemplary mathematics teacher encourages students to use varied methods of solving as long as it is the easiest way for them to arrive at the correct answer.*
16	*An exemplary mathematics teacher involves students in experiential learning and provides them with direct experiences to help them better retain concepts and information.*
18	*An exemplary mathematics teacher has patience in handling students with varied attitudes toward learning mathematics.*

2.5 Data Gathering Procedure

Data were sourced through two stages. Table 2 shows a simplified presentation of the two phases of data gathering, together with the sources of data, number of respondents or participants, and sampling procedure.

Table 2. A summary of the procedures, data source, respondents, and sampling procedure of the study

Phase Number	Data Source	Respondents/Participants	Sampling Procedure
1	Focus Group Discussion (FGD)	16 students	Purposive
1	Key Informant Interview (KII)	Ten mathematics teachers and one mathematics supervisor	Purposive
2	Survey	416 students	Proportionate Stratified Random Sampling

Data were gathered through the FGDs and KIIs to generate the attributes of an exemplary mathematics teacher. An FGD was conducted in two secondary schools in Davao del Sur. Junior High School students participated in this study. The result of the discussion was the validated attributes of a mathematics teacher who portrays exemplary practices to help alleviate students' difficulty in the subject. The KIIs were done separately with mathematics teachers and mathematics supervisors. They were individually subjected to a semi-structured interview where they were free to express their answers. Semi-structured interviewing is best for establishing a connection with participants before proceeding to more formal interviews, and it is suited for people not used to formal interviews (Tudy, 2017).

Transcripts were created for both responses of FGDs and KIIs. The researcher carefully listened to the recordings and jotted down the verbatim responses and conversations. Each transcript was read several times to understand the whole content. After completing the transcripts, significant statements and phrases were extracted from each transcript. Significant statements in each transcript were organized in a table with its corresponding location. After that, the researcher contextualized the verbatim responses by having the formulated meanings (Tudy, 2017). The second part of data gathering involved implementing the validated and pilot-tested survey questionnaire, the items of which were based on the attributes identified through the FGDs.

2.6 Data Analysis

To analyze the qualitative data of this study, the researcher employed the first three steps of Collaizi's (1978) strategy of phenomenological analysis. Colaizzi's process for phenomenological data analysis was used, as cited by Shosha (2012). Firstly, each transcript was carefully analyzed to formulate a meaningful gist of the procedure. Secondly, significant statements pointing to the study's phenomenon were extracted using each transcript. A document must stipulate these statements, and their pages and line numbers must be carefully noted. Thirdly, formulated meanings from these significant statements were constructed. The formulated meanings served as the items of the survey questionnaire (Tudy, 2017).

The results from the questionnaires were utilized to find the latent variables or factors among observed factors using Exploratory Factor Analysis (EFA) (Watkins, 2021). This deals with whether the covariance or correlations between a set of observed variables can be explained in a smaller number of observed constructs called latent variables or common factors. The aim of using this tool was the orderly simplification of the intercorrelated measures of the constructs of this study.

The contribution of EFA in this study was grouping the variables with similar characteristics. A correlation matrix was generated for all the variables. Then, factors were extracted from the correlation matrix based on the correlation coefficients of the variables. Lastly, factors were rotated to maximize the relationship between variables and some of the factors. Using the Structural Equation Model (SEM), Path Analysis evaluated the assumed causation among dependent and independent constructs (Owolabi et al., 2020). SEM estimated the shared measurement error for the constructs. The Chi-square value was used to determine the Model fit, supported by the Tucker-Lewis Index (TLI) and Comparative Fit Index (CFI) values.

After the Model's goodness of fit was satisfied, further analysis was done to evaluate the estimates and fit indices of the path diagram, which measured the contribution of the specified variables in the Model. Data obtained from EFA was inserted into AMOS to identify the best Model by examining the validity and reliability of the measures and several fit indices. Table 3 shows a summary of the data analysis utilized in the study.

Table 3. Data analysis used in the qualitative and quantitative phases of the study

Type of Phase	Statistical Tool/Analysis	Initial Output	Final Output
Qualitative	Collaizi's strategy	Formulated Meanings	Survey Questionnaire
Quantitative	Exploratory Factor Analysis (EFA), Confirmatory Factor Analysis (CFA) and Sequential Equation Modeling	Factors/Attributes of Exemplary Mathematics Teaching	Model/Framework

2.6 Ethical Considerations

The researcher highly considered the welfare of the participants. A permission letter was submitted to the school principals of all identified schools and to the teachers who were randomly chosen to be asked to participate voluntarily in this study. The purpose of the study will be explained to the participants. The participants were informed of the benefits and risks they could have while participating in the study. Participants were informed that they could be open with their responses, change or add answers whenever they felt the need to do so, and withdraw whenever they felt uncomfortable with the interview process.

The participants were informed of the confidentiality of their identities and responses to the interview questions. This was done by assigning each participant an identity code instead of using their actual names. Permission was requested from the participants to audiotape the interview. All information obtained during the interview was secured in a safe cabinet and can only be accessed by the researcher. After all relevant data were extracted, all the audio recordings were deleted. After all the data had been displayed, the researcher performed a member validation method by informing the participant about the accuracy of the transcription so that the participant could make necessary corrections to make his final standpoint in all the questions asked to him or her.

3.0 Results and Discussion

3.1 Development of the Survey Instrument

Table 4 provides examples of the significant statements identified and extracted after the FGDs and KIIs were conducted. These statements were written on separate sheets and coded based on their "transcript, page, and line numbers." Meanings were formulated from the significant statements.

Table 4. Examples of significant statements from FGDs and KIIs

Significant Statements	Transcript No.	Page No.	Line No.
"He/she should be well-equipped with the skills (e.g., mathematical skills), have in-depth knowledge, and know what he/she is talking about. The teacher cannot give inputs he/she does not have."	1	1	6-9
"Observes consistency when teaching; will not proceed unless (students) have already grasped an understanding of the concepts."	12	2	52-53

Table 5 provides examples of how significant statements were converted into formulated meanings. An expert researcher then checked the statements and their meanings, finding the process correct and the meanings consistent. The formulated meanings were utilized as the items in the initial instrument developed by the researcher.

Table 5. Examples of formulated meanings from significant interview statements

Significant Statements	Formulated Meanings
"There were times that I should meet them (the students) where they are, especially that we should focus on our students in our school."	Makes adjustments to the student's level of thinking and understanding
"The teacher should know how to modify and simplify complex topics."	Knows how to modify and simplify complex topics
"He/she (the teacher) gave a technique to simplify the concept easily and quickly, which helped me understand. The teacher gave a name to that technique."	Teaches students shortcuts and techniques for solving

3.2 Underlying Attributes of an Exemplary Mathematics Teacher on the Perspective of Students

The data from the survey were subjected to data reduction to determine students' perspectives on the attributes of an exemplary Mathematics teacher at the junior high school level. First, an exploratory factor analysis (EFA) was conducted for the attributes of exemplary mathematics teachers, and it included 65 items. This analysis aimed to reduce data by removing highly correlated items and grouping uncorrelated items into underlying or latent relationships. Several tests were performed to identify the number of factors, including Kaiser-Meyer-Olkin (KMO) criterion, scree plot of eigenvalues, and other analyses. KMO is used to measure sampling adequacy. In the first run of EFA, the results yielded a KMO value of .971, which is within the acceptable value, as shown in Table 6. Kaiser (1974) recommended accepting values more significant than .5 as barely acceptable. It means that the sample size qualifies for factor analysis. Bartlett's Test of Sphericity with a p-value of .000 means at least one significant correlation between two items. Table 7 shows that there were eight factors and a 63.694% total Variance explained, higher than the 60% threshold (Hair et al., 2012). Initially, Oblimin Rotation was used to determine if the factors were orthogonal or oblique. The Component Correlation Matrix showed no coefficients greater than 0.5, meaning the factors were not strongly correlated. Hence, a Varimax Rotation was applied.

Table 6. KMO and Bartlett's test result

KMO and Bartlett's Test		
Kaiser-Meyer-Olkin Measure of Sampling Adequacy		0.971
Bartlett's Test of Sphericity	Approx. Chi-Square	20770.139
	Df	2080
	Sig.	0.000

After determining the factors and their corresponding items, a reliability test was conducted to determine the reliability of the new factors. Results revealed that Factor 1 got a Cronbach Alpha of .969, Factor 2 with .963, Factor 3 with .901, and Factor 4 with .883. Factors 5, 6, 7, and 8 have only one item loaded each, so these were not considered final components. A single item will have issues in AMOS regarding validity checking. A minimum of three items is needed to check the validity of AMOS. Moreover, researchers have suggested varying items per factor ranging from three to five to represent each factor (MacCallum et al., 1999; Raubenheimer, 2004). The factor and related items are removed if a factor has less than three loadings (Barkley & Burns, 2000; Costello & Osborne, 2005).

Table 7 shows the analysis results, which yielded eight factors and explained the equivalent percentage of Variance.

Table 7. Principal components analysis summaries: eigenvalues and percent of Variance explained

Factor	Eigenvalue	% of Variance	Cumulative %
1	30.355	46.699	46.699
2	3.110	4.784	51.483
3	2.014	3.098	54.582
4	1.468	2.259	56.840
5	1.254	1.929	58.769
6	1.140	1.753	60.522
7	1.046	1.609	62.131
8	1.016	1.563	63.694

The first factor, including 23 items, was labeled "Expertise" with an eigenvalue of 30.355 and explained 46.70% of the total Variance. The second factor, including 19 items, was labeled "Effective Communication with Students" with an eigenvalue of 3.110 and explained 4.78% of the total Variance. The third factor, including nine items, was labeled "Devolution" with an eigenvalue of 2.014 and explained 3.10% of the total Variance. The last factor, including nine items, was labeled "Teacher's Disposition" with an eigenvalue of 1.468 and explained 2.26% of the total Variance. Each of these factors will be discussed in the succeeding section.

3.3 Underlying Attributes of an Exemplary Mathematics Teacher

Factor 1, which the researcher labeled "Expertise," pertained to teachers' content knowledge (e.g., Items 24, 42, 32, 31, 23, 37, 44, 27, and 33). Students in this study are expecting teachers to have mastered the content. But apparently, they should also know how to deliver this topic to students using various techniques (e.g., Items 28, 43, 36, 39, 38, 30, and 45) and the right pacing (e.g., Items 26 and 22). Factor 1 also pertained to understanding the student's level of thinking and developing into students the other side of mathematics, which is doing it with fun and motivation (e.g., Items 29, 46, 41, 40, and 35).

The second factor, which was labeled as "Effective Communication with Students," pertained to teachers' way of knowing the students' learning capability through interactions (e.g., Items 54, 58, 57, 50, 53, 64, 47, and 55). In a way, teachers should carefully know the students and the specific strategy to employ to meet their particular level of understanding (e.g., Items 56, 65, 51, 52, 48, 62, 60, and 61) and know how to boost students confidence in learning the subject (e.g., Items 49, 59, and 63). In this study, the students prefer teachers who always anticipate what students are thinking and what things are confusing for them. Teachers need to know the strategies they employ and whether these are fitted to the student's perspective of learning. Teachers cannot just have a group activity when learners are not ready. In short, any step to be taken should be patterned on the learning status of the students.

Some of the essential keys found in this study are integrating new ways of dealing with mathematical problems, imposing a positive learning environment for students, considering close interaction with students, not only pure transmission of content, adopting the attitude of learners, and making new techniques as part of the repertoire. In this study, students assessed an excellent mathematics teacher as someone who teaches well and initiates a positive teacher-student relationship through open Communication. Teachers are mandated to prioritize the students' level of understanding of the subject. It is not just firmly teaching them the learning competencies but also giving them the liberty of how they will learn the subject matter. Ultimately, it's always a reality that the students can directly vouch for whether a teacher is best in his field. Communicating with the students may be simple in words, but it is an intricate role for a teacher. Profiling cannot suffice the purpose of knowing the students well. Instead, knowing their engagement in the teaching-learning process may merit a good relationship between teacher and students. A teacher can extensively show mathematical competence in the class, but this could become trash when students feel uneasy about staying in the class because of difficulty and boredom. Letting them know how to get x and y is something terrifying and tiring for them. Hence, creating meaning in every task for students is bound to be significant.

The third factor, labeled as "Devolution," pertained to making students realize and feel their full responsibility and liability for the learning imparted to them (e.g., Items 14, 15, 16, 17, 19, and 21). But apparently, students still need assistance and monitoring from the teacher (e.g., Items 20, 13, and 12) to achieve the expected learning outcome. Teachers no longer take lectures but provide students with tasks they can experience meaningfully. Teachers need to foresee the emerging and inchoate thinking of the students. Students might be unable to show the same attitudes toward the subject, but they mainly value contextual learning. Traditional lectures might be obscure for them. Writing the content and formula on the board might be too dull for the kind of students in a class. Talking all the time without asking students' opinions might be too dominating for them. These are the things that a teacher must consider. Entrusting the learning to students may greatly help, but the teacher facilitates the whole process.

The final factor is labeled as "Teacher's Disposition" since most of the items are about a teacher who provides enlightenment for students who are struggling the most (e.g., Items 3, 4, 10, and 8) and who is optimistic enough in making these students love the subject, a teacher who has passion and heart in motivating these type of students, and a teacher who can survive the daily struggles with good humor, a pleasant disposition, and staying power (e.g., Items 1, 2, and 18). Students in this study viewed good mathematics teachers as someone who can transcend beyond the four corners of the classroom. Hearing students' thoughts about the subject itself could help stretch standards. The students mostly want the warmth of teaching (e.g., Items 6 and 7). A teacher cannot comment on students' mistakes in solving but let them feel that their mistakes could be the springboard for a correct conception of the topic. Teachers must let students feel they are greatly appreciated in everything they do.

Other studies did not discuss the abovementioned factors, but having these factors in one framework could be different. The table in the review of related studies would be a good reference for a deeper discussion of each factor. Table 8 shows what factor/s are associated with the dimensions discussed in the various frameworks of good mathematics teaching. Also, this shows the connection of the four-factor model to the existing literature.

Table 8. Literature included frameworks of exemplary mathematics teaching

Researches	CPK	Lc	ID	WD	KoA
Kaur (2008, 2009)	F1			F4	F2
Murray (2011)	F1	F2		F4	
White, Barnes, Lawson, and Johnson (2009)	F1	F2	F3	F4	F2
MATHTED (2011)	F1	F2	F3	F4	
Latterel (2008)	F1	F2	F3		F2
Leong (2014)	F1	F2	F3	F4	
Schoenfeld and Kilpatrick (2008)	F1	F2	F3	F4	
Gonzales (2020)	F1	F2	F3	F4	

Note: *Knowledge (CPK), Learner-centered (Lc), Instruction Differentiation (I.D.), Warmth Disposition, Knowledge of Assessment (KoA)*
****F1 (Expertise), F2 (Effective Communication with Students, F3 (Devolution), F4 (Teacher's Disposition)*

Based on the table above, three out of seven frameworks did not mention all the factors of this study. Kaur's (2008, 2009) study mentioned only Factors 1 and 4. Factor 3 was not consistent with the study of Murray (2011). Factor 4 was not discussed in Latterel's (2008) study. On the other hand, these factors not mentioned in the other studies were factors of the current Model of exemplary mathematics teaching. This implies that this four-factor model covers all dimensions of good mathematics teaching. The Knowledge on Assessment could be integrated with Factor 2, knowing that a teacher cannot give an appropriate assessment task if he/she doesn't know his/her students' learning capability and doesn't know how to communicate the level of performance to them. Factor 2 in Kaur's (2009) study was consistent only with Knowledge of Assessment but not with the Learner-Centered Approach.

Factor 1, labeled as "Expertise," was consistent with the component of the three frameworks (Murray, 2011; Kaur, 2009; and White et al., 2009) in terms of content knowledge of teachers. Murray's (2011) "explaining so that students understand" concurrently focused on the expertise of teachers in elaborating the content. But in this study, the capacity of the teacher to explain is just part and parcel of the first factor. In this aspect, Kaur (2009) also highlighted a good mathematics teacher as the best at explaining a topic. Deliberately, "explaining well" is the first attribute in the framework of White et al. (2009). The frameworks of the prior studies commonly considered Content and Pedagogical Knowledge (CPK) as a component, not just the "explaining component" but also the manner of delivery of instruction to students was given emphasis.

The second factor, "effective communication with students," was articulated in all prior studies except in the study of Kaur (2009). Kaur's (2009) framework focused on the teacher's mathematical competence in facilitating student learning rather than setting students at the center of every teaching detail. The current study highlights the importance of knowing students well through interactions to implement appropriate teaching strategies and interactive activities. Similarly, the same result is mirrored in the study of Murray (2011), which is the "one-on-one help and recognizing individual differences" component. A teacher's warmth in teaching should radiate to students to instill motivation and love for the subject.

Factor 3, labeled "Devolution," was consistent with five of seven prior studies. The concept of devolution in teaching mathematics was not mentioned in the frameworks of Kaur (2009) and Murray (2011). However, in this current framework, the students mostly favored a good mathematics teacher who can delegate time to craft differentiated activities wherein they can experience things in a natural context and feel responsible for their learning.

The fourth factor, "Teacher's Disposition," was consistent with all frameworks except for Latterel's (2008) model of mathematics teaching. Compared to the study of Leong (2014), the first teaching model, which implies the excellent personality of a mathematics teacher, can be associated with the attributes of this factor. The fourth factor can be viewed along with the study of Davis (2003), which exclaimed that a positive relationship with students is having emotional closeness, warmth, caring, support, acceptance, respect, fairness, and low levels of conflict and dependency as learning is being pushed through is a great help for students. The same result was expounded in the study of Alsaleh et al. (2019) that students must receive the utmost care intended for their learning and that an ideal mathematics teacher must support students with positive mathematical identities. To support this, Kaur (2008) has stressed in his study that the best mathematics teacher is patient, understanding, caring, good at the content, explains explicitly, ensures that all students clearly understand the lesson, assists when students encounter signs of difficulty and provides prompt feedbacks on their learning status. Perhaps students in this study view an excellent mathematics teacher as someone who can shift from firm to gentle and knows how to blend in with their attitudes towards the subject.

In a nutshell, this study's findings were supported by the cited literature. This implies that the four attributes of exemplary mathematics teaching (Expertise, Effective Communication with Students, Evolution, and Teacher's Disposition) can be fully explained using the established body of knowledge. Hence, these factors constitute high reliability in developing the structural Model of an exemplary mathematics teacher.

3.4 A Structural Model of an Exemplary Mathematics Teacher in Junior High School Level

Confirmatory Factor Analysis (CFA) was performed on the hypothesized Model containing 60 observed variables with four latent variables using Analysis of Moment Structure (AMS) version 21. Data obtained from EFA was inserted into AMOS to identify the best Model by looking into the validity and reliability of the measures and several fit indices. The maximum likelihood was used for the estimation, including minimization history, standardized estimates, residual moments, and modification indices. After model specification, an acceptable level of fit was achieved.

Given the large sample size, the chi-square was very large at 364.580, which was understandable. Also, the other indices showed the following values- CMIN / df = 3.143, GFI = 0.889, RMSEA = 0.078, PCLOSE = 0.000, NFI = 0.902, CFI = 0.931, TLI = 0.919, CFI = 0.931, PCFI = 0.794 and PNFI = 0.769. These values suggested that the data did not fit well with the hypothesized Model. Also, there were very high covariance values in the modification index between some of the observed variables.

With the goodness of fit not achieved, re-specification was needed for the hypothesized Model based on the criteria set by Hair *et al*. (2006):

1. Review the factor loading of each item, where it must exceed 0.50, and remove the items that do not meet this criterion.
2. Review of the standardized residual covariances where the items with a value of more than two will be dropped.
3. Review the modification index (d≤20) to improve the Model.

All factor loadings of the items were significant and varied between .59 and .90. No items (variables) were found to have less than .50 loadings. To obtain Model fit as well as sufficient discriminant and convergent validity, 18 problematic items with standardized residual covariances of more than two were removed (A2, A14, A56, A54, A57, A50, A48, A22, A33, A27, A30, A39, A43, A21, A52, A55, A23, and A40). Lastly, a look into the modification index suggested drawing covariances (double-headed arrow) for variables with high values. Hence, covariances were drawn as suggested by the modification index to improve the model.

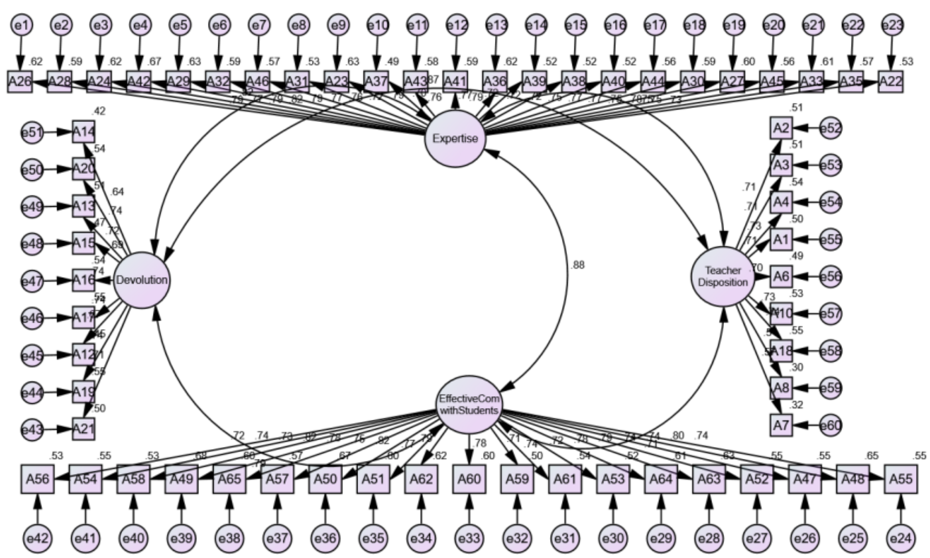

Figure 1. Hypothesized model

After the specification process, the modified Model was tested again, and the results showed the Model's goodness of fit. All the loading values were more than 0.5. Moreover, the goodness of fit indexes satisfied the threshold requirements- CMIN / df = 2.000, RMSEA = 0.049, CFI = 0.934, TLI = 0.928, PCFI = 0.866, and PNFI = 0.813. The GFI (0.848) and NFI (0.877) were slightly lower but still within acceptable values. With so many observed variables in the Model, it is almost impossible to satisfy perfection. These indices meant there was an excellent fit to the observed data. Also, the paths between latent variables were significant. Thus, this modified Model established the relationship among the factors (latent variables). Moreover, all the paths from the latent and observed variables (question items) were also significant. Hence, the measurement model (observed variables to latent variables) and the structure model (relationship among latent variables) were established.

3.5 Testing the Validity and Reliability

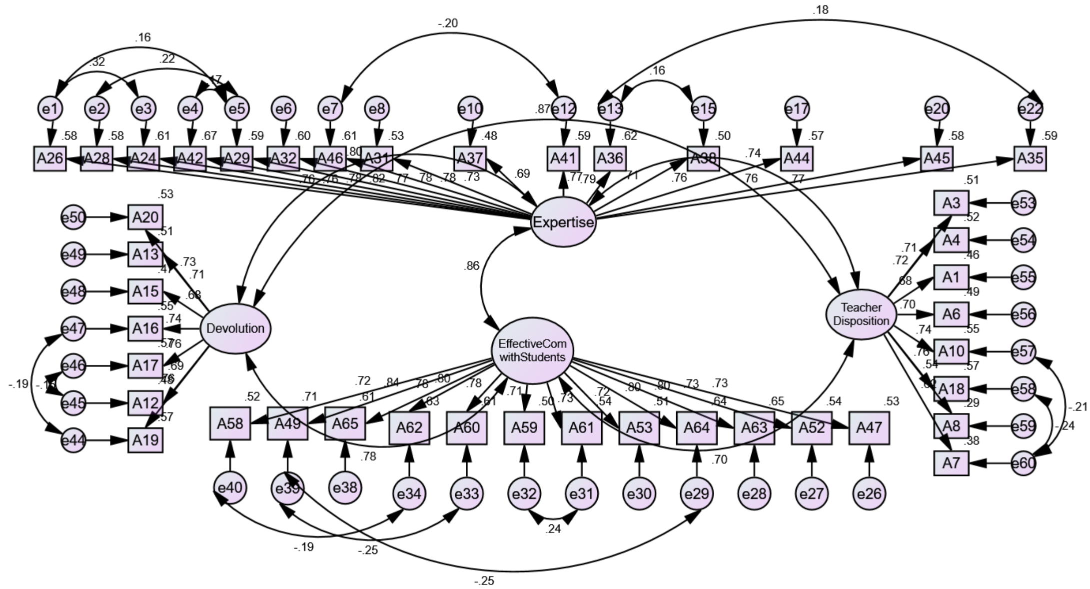

Figure 2. Modified Model (standardized Estimates)

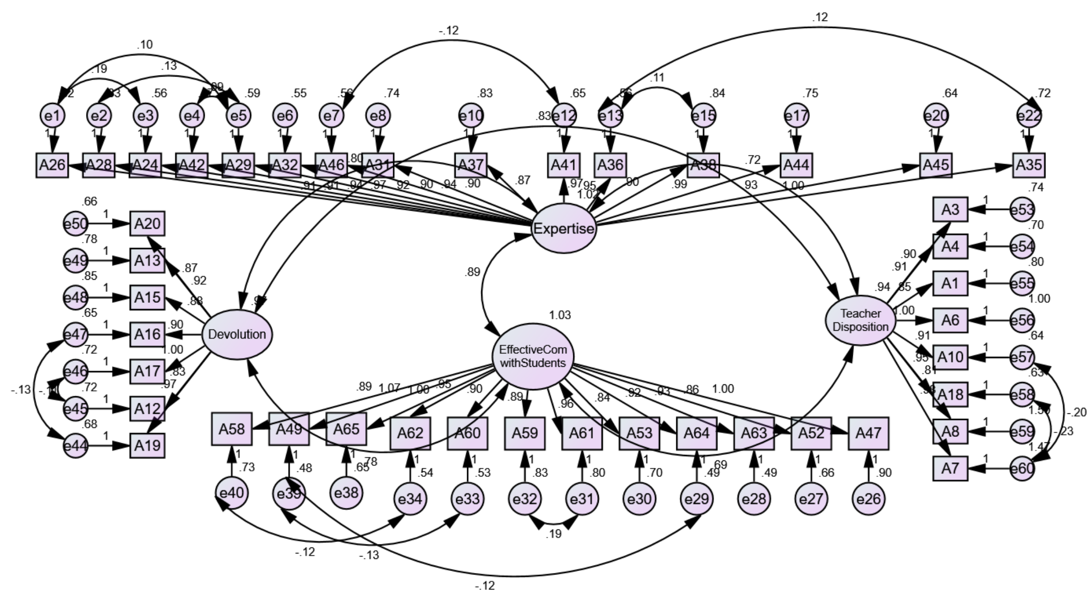

Figure 3. Modified Model (Unstandardized Estimates)

Tables 9a, 9b, 9c, and 9d show the unstandardized regression estimates, which suggested that all the paths of each factor (Expertise, Effective Communication with Students, Devolution, and Teacher's Disposition) were significant, as confirmed by the significant p-values. This means that all items significantly contribute to each factor, confirming the Model's validity.

Table 9a. Unstandardized regression estimates of the first factor

Items	Estimate	S.E.	C.R.	P
Expertise				
A26	0.914	0.055	16.667	***
A28	0.915	0.055	16.618	***
A24	0.935	0.054	17.241	***
A42	0.973	0.053	18.259	***
A29	0.917	0.054	16.867	***
A32	0.899	0.053	17.023	***
A46	0.936	0.054	17.211	***
A31	0.897	0.057	15.752	***
A37	0.869	0.058	14.915	***
A41	0.966	0.057	16.877	***
A36	0.950	0.049	19.259	***
A38	0.902	0.059	15.224	***
A44	0.992	0.060	16.545	***
A45	0.930	0.056	16.679	***
A35	1.000			

Table 9b. Unstandardized regression estimates of the second factor

Items	Estimate	S.E.	C.R.	P
Effective Communication with students				
A47	1.000			
A52	0.865	0.057	15.108	***
A63	0.932	0.056	16.714	***
A64	0.918	0.056	16.505	***
A53	0.844	0.057	14.733	***
A61	0.955	0.063	15.144	***
A59	0.892	0.062	14.509	***
A60	0.902	0.056	16.174	***
A62	0.949	0.058	16.483	***
A65	1.004	0.062	16.230	***
A49	1.070	0.061	17.501	***
A58	0.886	0.059	14.902	***

Table 9c. Unstandardized regression estimates of the third factor

Items	Estimate	S.E.	C.R.	P
Devolution				
A19	0.826	0.062	13.366	***
A12	1.000			
A17	0.900	0.059	15.333	***
A16	0.875	0.062	14.116	***
A15	0.917	0.062	14.848	***
A13	0.874	0.058	15.117	***
A20	0.826	0.062	13.366	***

Table 9d. Unstandardized regression estimates of the fourth factor

Items	Estimate	S.E.	C.R.	P
Teacher's Disposition				
A3	0.902	0.066	13.563	***
A4	0.910	0.066	13.791	***
A1	0.853	0.066	12.952	***
A6	1.000			
A10	0.910	0.065	14.000	***
A18	0.954	0.067	14.290	***
A8	0.809	0.078	10.374	***

For the covariances between latent variables, as shown in Table 10 below, each factor was significantly related to each other, meaning the four factors were internally consistent. This means these factors agree to measure a broader attribute: the attributes of an exemplary mathematics teacher in Junior High School.

Table 10. Covariances of the latent variables

			Estimate	S.E.	C.R.	P
Expertise	<-->	Devolution	0.799	0.080	9.935	***
Expertise	<-->	Teacher's Disposition	0.723	0.079	9.212	***
Effective Communication with Students	<-->	Devolution	0.779	0.081	9.616	***
Effective Communication with Students	<-->	Teacher's Disposition	0.693	0.078	8.873	***
Expertise	<-->	Effective Communication with Students	0.888	0.087	10.156	***
Devolution	<-->	Teacher's Disposition	0.832	0.086	9.725	***

4.0 Conclusions

Students' depictions of exemplary mathematics teachers in Junior High School circulate the four attributes (Expertise, Knowledge of Students, Devolution, and Mathematics Disposition). Students view success in learning mathematics when teachers are equipped with these four attributes. The participants of this study considered an excellent mathematics teacher to possess expertise in content and pedagogical knowledge. A good mathematics teacher understands everything about students in mathematics. A teacher who places students as the center of learning, giving them the whole opportunity to discover and learn things meaningfully, is considered exemplary in teaching. Lastly, a teacher who can devote passion and love to teaching the subject and make students feel comfortable learning by creating a positive learning environment is also labeled as an exemplary mathematics teacher.

5.0 Contributions of Authors

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Research is not funded since it is an academic requirement.

7.0 Conflict of Interests

No conflict of interest with any other person or institution.

8.0 Acknowledgment

Not applicable

9.0 References

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