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A Case Study on the Static Behavior of a Pedestrian Bridge Built with a Steel Pratt Truss Deck Bridge Supported by an Inclined Roller Support

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Abstract. Construction of bridges commonly uses trusses due to their lightweight design, which can endure heavy loads and span large distances. However, there are instances when the supports of these trusses do not lie normally on a horizontal surface. For some, the supports lie on inclined foundations, which is the focus of this study. The main objective of this study is to present the basic theory of plane truss analysis with an inclined roller support using the Direct Stiffness Method (DSM) in comparison to the use of the Graphic Rapid Analysis Software Program (GRASP) to analyze the static behavior of the structure. Furthermore, a case study is conducted to check the effects of inclined supports on a plane truss structure. Based on the results, it was discovered that the introduction of inclined roller supports does have a negative effect on the stability of the structure.

Keywords: Bridge; Bridge support; Inclined roller support; Plane truss; Graphic rapid analysis software; Program; Direct stiffness method.

1.0 Introduction

Many structures are being built with the use of trusses, whether it be made of steel or wood. The primary advantage of trusses is that it can cover large spans and support heavy loads. Bridges and roof systems often use trusses because of their characteristics. The use of structural trusses entails analysis especially when it comes to the material properties used as well as the length of each member used. Each element is designed such that it will be able to support a certain number of loads without noticeable deflections. The most important property being considered in the analysis of trusses is the stiffness of each element. However, there are instances when these trusses are supported by inclined supports, which may affect the load capacity of the truss system. The study will present the basic theory of analyzing Plane Trusses with the use of the Direct Stiffness Method (DSM) and the use of Graphic Rapid Analysis Software Program (GRASP) to compare the static behavior of the structure. The study will also present a case study of a pedestrian bridge built using a Pratt truss deck bridge having various angles of inclination of the roller support to observe how the inclination would affect the structure.

2.0 Structural Model and Matrix Formulation

2.1 Structural Model

In Figure 1, a 9-element structure was used to demonstrate the analysis of a plane truss with a hinge support at one end, and a roller support inclined by 30 degrees at the other end using the Direct Stiffness Method (DSM). Loads were applied at both horizontal and vertical directions of the joints, specifically, at joints where there are no restraints. The following are the material properties of the truss elements used in the structure:

Modulus of Elasticity: 200,000 MPa
Area of Members: 2,813.42 mm^2
Moment of Inertia: 19.55 x 10^6 mm^4

In Figure 2, the global axes for each node were established. In Figure 3, the truss elements were disassembled to display the local axes for each element. To simplify the computation using DSM, the element properties were displayed in Table 1.

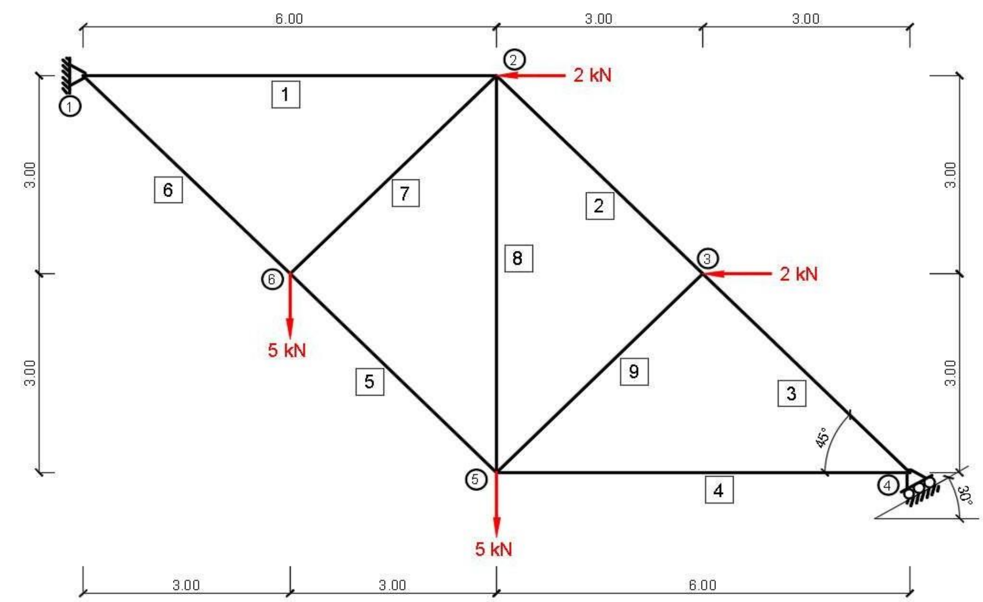

Figure 1. Plane truss model with inclined roller support

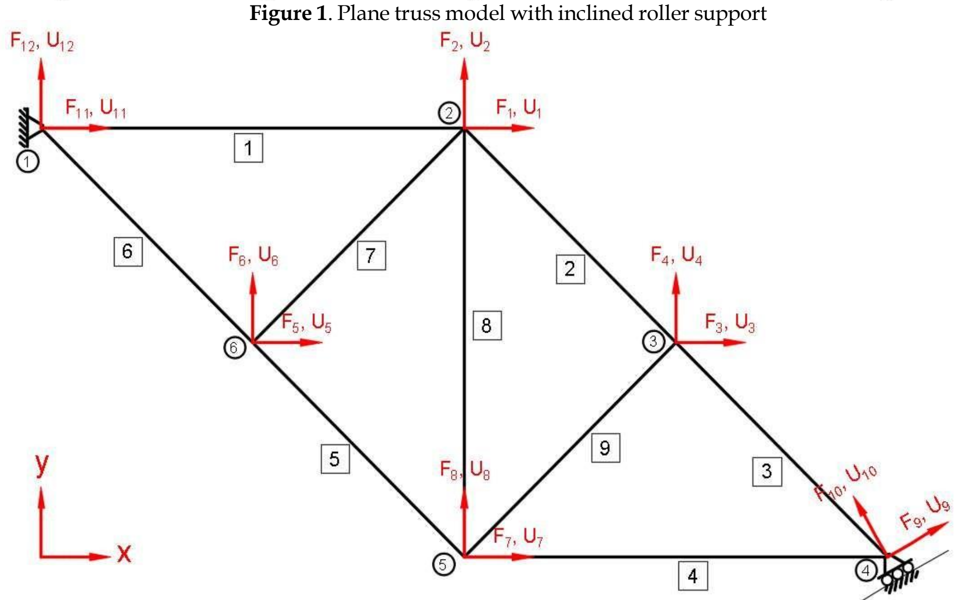

Figure 2. Global Axes

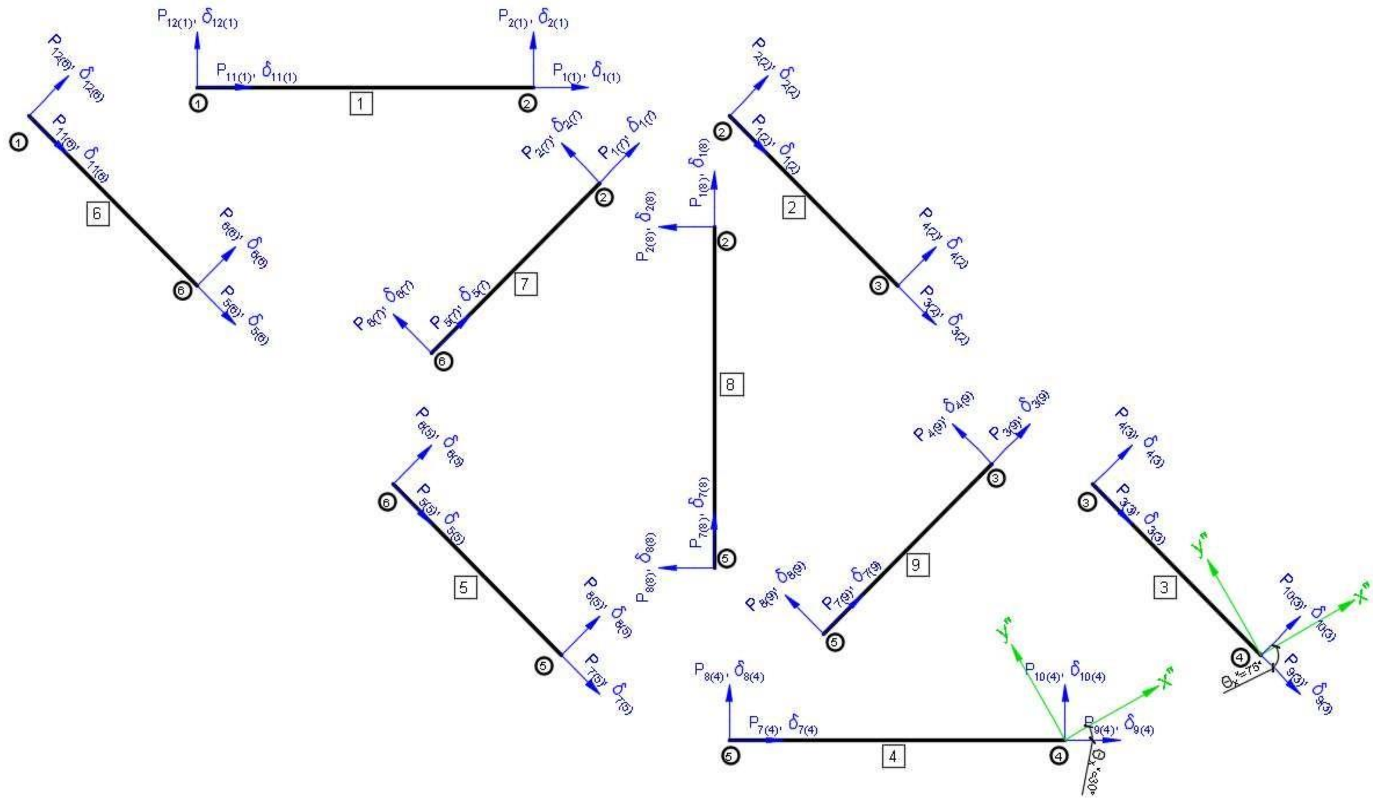

Figure 3. Local axes of each truss element

ELEMENT PROPERTIES											
Element	Node	Length	E	A	EA	θ	cos θ	sin θ	θx"	cos θx"	sin θx"
		(m)	(Mpa)	(mm²)	kN	(deg)			(deg)		
1	1 → 2	6.00	200,000	2,813.42	562,683.56	0.00	1.000	0.000			
2	2 → 3	4.24	200,000	2,813.42	562,683.56	-45.00	0.707	-0.707			
3	3 → 4	4.24	200,000	2,813.42	562,683.56	-45.00	0.707	-0.707	-75.00	0.259	-0.966
4	5 → 4	6.00	200,000	2,813.42	562,683.56	0.00	1.000	0.000	-30.00	0.866	-0.500
5	6 → 5	4.24	200,000	2,813.42	562,683.56	-45.00	0.707	-0.707			
6	1 → 6	4.24	200,000	2,813.42	562,683.56	-45.00	0.707	-0.707			
7	6 → 2	4.24	200,000	2,813.42	562,683.56	45.00	0.707	0.707			
8	5 → 2	6.00	200,000	2,813.42	562,683.56	90.00	0.000	1.000			
9	5 → 3	4.24	200,000	2,813.42	562,683.56	45.00	0.707	0.707			

Table 1. Element properties

2.2 Matrix Formulation

Using the Direct Stiffness Method, the local stiffness matrices for each truss element were constructed based on the given properties. Likewise, the transformation matrices were constructed based on the established local axes of the truss elements.

Element Local Stiffness Matrix

Tables 2 to 10 display the local stiffness matrix (k) for each element.

	11	12	1	2	
	0.167	0.000	-0.167	0.000	11
k_1 = EA	0.000	0.000	0.000	0.000	12
	-0.167	0.000	0.167	0.000	1
	0.000	0.000	0.000	0.000	2

Table 2. Local stiffness matrix of member 1

	1	2	3	4	
	0.236	0.000	-0.236	0.000	1
k_2 = EA	0.000	0.000	0.000	0.000	2
	-0.236	0.000	0.236	0.000	3
	0.000	0.000	0.000	0.000	4

Table 3. Local stiffness matrix of member 2

	3	4	9	10	
	0.236	0.000	-0.236	0.000	3
k_3 = EA	0.000	0.000	0.000	0.000	4
	-0.236	0.000	0.236	0.000	9
	0.000	0.000	0.000	0.000	10

Table 4. Local stiffness matrix of member 3

	7	8	9	10	
	0.167	0.000	-0.167	0.000	7
k_4 = EA	0.000	0.000	0.000	0.000	8
	-0.167	0.000	0.167	0.000	9
	0.000	0.000	0.000	0.000	10

Table 5. Local stiffness matrix of member 4

	5	6	7	8	
	0.236	0.000	-0.236	0.000	5
k_5 = EA	0.000	0.000	0.000	0.000	6
	-0.236	0.000	0.236	0.000	7
	0.000	0.000	0.000	0.000	8

Table 6. Local stiffness matrix of member 5

	11	12	5	6	
	0.236	0.000	-0.236	0.000	11
k_6 = EA	0.000	0.000	0.000	0.000	12
	-0.236	0.000	0.236	0.000	5
	0.000	0.000	0.000	0.000	6

Table 7. Local stiffness matrix of member 6

	5	6	1	2	
	0.236	0.000	-0.236	0.000	5
k_7 = EA	0.000	0.000	0.000	0.000	6
	-0.236	0.000	0.236	0.000	1
	0.000	0.000	0.000	0.000	2

Table 8. Local stiffness matrix of member 7

	7	8	1	2	
	0.167	0.000	-0.167	0.000	7
k_8 = EA	0.000	0.000	0.000	0.000	8
	-0.167	0.000	0.167	0.000	1
	0.000	0.000	0.000	0.000	2

Table 9. Local stiffness matrix of member 8

	7	8	3	4	
	0.236	0.000	-0.236	0.000	7
k_9 = EA	0.000	0.000	0.000	0.000	8
	-0.236	0.000	0.236	0.000	3
	0.000	0.000	0.000	0.000	4

Table 10. Local stiffness matrix of member 9

Element Transformation Matrix

Tables 11 to 19 display the transformation matrix (β) for each element.

	11	12	1	2	
	1.000	0.000	0.000	0.000	11
β_1 =	0.000	1.000	0.000	0.000	12
	0.000	0.000	1.000	0.000	1
	0.000	0.000	0.000	1.000	2

Table 11. Transformation matrix of member 1

	1	2	3	4	
	0.707	-0.707	0.000	0.000	1
β_2 =	0.707	0.707	0.000	0.000	2
	0.000	0.000	0.707	-0.707	3
	0.000	0.000	0.707	0.707	4

Table 12. Transformation matrix of member 2

	3	4	9	10	
	0.707	-0.707	0.000	0.000	3
β_3 =	0.707	0.707	0.000	0.000	4
	0.000	0.000	0.259	-0.966	9
	0.000	0.000	0.966	0.259	10

Table 13. Transformation matrix of member 3

	7	8	9	10	
	1.000	0.000	0.000	0.000	7
β_4 =	0.000	1.000	0.000	0.000	8
	0.000	0.000	0.866	-0.500	9
	0.000	0.000	0.500	0.866	10

Table 14. Transformation matrix of member 4

	5	6	7	8	
	0.707	-0.707	0.000	0.000	5
β_5 =	0.707	0.707	0.000	0.000	6
	0.000	0.000	0.707	-0.707	7
	0.000	0.000	0.707	0.707	8

Table 15. Transformation matrix of member 5

	11	12	5	6	
	0.707	-0.707	0.000	0.000	11
β_6 =	0.707	0.707	0.000	0.000	12
	0.000	0.000	0.707	-0.707	5
	0.000	0.000	0.707	0.707	6

Table 16. Transformation matrix of member 6

	5	6	1	2	
	0.707	0.707	0.000	0.000	5
β_7 =	-0.707	0.707	0.000	0.000	6
	0.000	0.000	0.707	0.707	1
	0.000	0.000	-0.707	0.707	2

Table 17. Transformation matrix of member 7

	7	8	1	2	
	0.000	1.000	0.000	0.000	7
β_8 =	-1.000	0.000	0.000	0.000	8
	0.000	0.000	0.000	1.000	1
	0.000	0.000	-1.000	0.000	2

Table 18. Transformation matrix of member 8

	7	8	3	4	
	0.707	0.707	0.000	0.000	7
β_9 =	-0.707	0.707	0.000	0.000	8
	0.000	0.000	0.707	0.707	3
	0.000	0.000	-0.707	0.707	4

Table 19. Transformation matrix of member 9

3.0 Matrix Solution Using Excel

3.1 Computation of the Element Global Stiffness Matrix

Using the constructed local stiffness matrix (k) and transformation matrix (β), the global stiffness matrix [Ke] of each truss element can be computed using equation 1. Tables 20 to 28 display the computed global stiffness matrix [Ke] for each element.

$$[\boldsymbol{K_e}] = [\beta^{T}][k][\beta] \qquad \text{(Equation 1)}$$

	11	12	1	2	
	0.167	0.000	-0.167	0.000	11
$[K_1]$ = EA	0.000	0.000	0.000	0.000	12
	-0.167	0.000	0.167	0.000	1
	0.000	0.000	0.000	0.000	2

Table 20. Global stiffness matrix of member 1

	1	2	3	4	
	0.118	-0.118	-0.118	0.118	1
$[K_2]$ = EA	-0.118	0.118	0.118	-0.118	2
	-0.118	0.118	0.118	-0.118	3
	0.118	-0.118	-0.118	0.118	4

Table 21. Global stiffness matrix of member 2

	3	4	9	10	
	0.118	-0.118	-0.043	0.161	3
$[K_3]$ = EA	-0.118	0.118	0.043	-0.161	4
	-0.043	0.043	0.016	-0.059	9
	0.161	-0.161	-0.059	0.220	10

Table 22. Global stiffness matrix of member 3

	7	8	9	10	
	0.167	0.000	-0.144	0.083	7
$[K_4]$ = EA	0.000	0.000	0.000	0.000	8
	-0.144	0.000	0.125	-0.072	9
	0.083	0.000	-0.072	0.042	10

Table 23. Global stiffness matrix of member 4

	5	6	7	8	
	0.118	-0.118	-0.118	0.118	5
$[K_5]$ = EA	-0.118	0.118	0.118	-0.118	6
	-0.118	0.118	0.118	-0.118	7
	0.118	-0.118	-0.118	0.118	8

Table 24. Global stiffness matrix of member 5

	11	12	5	6	
	0.118	-0.118	-0.118	0.118	11
$[K_6]$ = EA	-0.118	0.118	0.118	-0.118	12
	-0.118	0.118	0.118	-0.118	5
	0.118	-0.118	-0.118	0.118	6

Table 25. Global stiffness matrix of member 6

	5	6	1	2	
	0.118	0.118	-0.118	-0.118	5
$[K_7]$ = EA	0.118	0.118	-0.118	-0.118	6
	-0.118	-0.118	0.118	0.118	1
	-0.118	-0.118	0.118	0.118	2

Table 26. Global stiffness matrix of member 7

	7	8	1	2	
	0.000	0.000	0.000	0.000	7
$[K_8]$ = EA	0.000	0.167	0.000	-0.167	8
	0.000	0.000	0.000	0.000	1
	0.000	-0.167	0.000	0.167	2

Table 27. Global stiffness matrix of member 8

	7	8	3	4	
	0.118	0.118	-0.118	-0.118	7
$[K_9]$ = EA	0.118	0.118	-0.118	-0.118	8
	-0.118	-0.118	0.118	0.118	3
	-0.118	-0.118	0.118	0.118	4

Table 28. Global stiffness matrix of member 9

3.2 Construction of Expanded Structure Global Stiffness Matrix

With the computed global stiffness matrix [Ke] of each element, the expanded structure global stiffness matrix [Ks] can be constructed by getting the summation of the element global stiffness matrices. Table 29 shows the expanded global stiffness matrix [Ks].

	1	2	3	4	5	6	7	8	9	10	11	12	
$[K_S]$ = EA	0.402	0.000	0	0	0	0	0.000	0.000	0	0	-0.167	0	1
	0.000	0.402	0	0	0	0	0.000	-0.167	0	0.000	0	0	2
	0	0	0.354	-0.118	0	0	-0.118	-0.118	0	0	0.000	0	3
	0	0	-0.118	0.354	0	0.000	-0.118	-0.118	0	0	0	0	4
	0	0	0	0	0.354	0	-0.118	0	0	0	0	0	5
	0	0	0	0.000	0	0.354	0	0	0	0	0	0	6
	0.000	0.000	-0.118	-0.118	-0.118	0	0.402	0	-0.144	0	0	0	7
	0.000	-0.167	-0.118	-0.118	0	0	0	0.402	0	0	0	0.000	8
	0	0	0	0	0	0	-0.144	0	0.141	0	0	0	9
	0	0.000	0	0	0	0	0	0	0	0.262	0	0	10
	-0.167	0	0.000	0	0	0	0	0	0	0	0.285	0	11
	0	0	0	0	0	0	0	0.000	0	0	0	0.118	12

Table 29. Expanded global structure stiffness matrix

3.3 Global Nodal Forces

Table 30 displays the known global nodal forces [FP] and unknown reactions at supports [FQ].

F (kN)		
F_P	F_1 =	-2.00
	F_2 =	0.00
	F_3 =	-2.00
	F_4 =	0.00
	F_5 =	0.00
	F_6 =	-5.00
	F_7 =	0.00
	F_8 =	-5.00
	F_9 =	0.00
F_Q	F_{10} =	?
	F_{11} =	?
	F_{12} =	?

Table 30. Global nodal forces and reactions at supports

3.4 Global Nodal Displacements

Table 31 displays the unknown global nodal displacements [UP] and zero nodal displacements at supports [UQ].

U (m)		
U_P	U_1 =	?
	U_2 =	?
	U_3 =	?
	U_4 =	?
	U_5 =	?
	U_6 =	?
	U_7 =	?
	U_8 =	?
	U_9 =	?
U_Q	U_{10} =	0.00
	U_{11} =	0.00
	U_{12} =	0.00

Table 31. Global nodal displacements

3.5 Global Structure Stiffness Equation

Using the expanded global structure stiffness matrix [KS] in Table 29, global nodal forces [F] in Table 30, and global nodal displacements [U] in Table 31, the global structure stiffness equation can be formulated using equation 2. Equation 3 displays the partitioned matrices for the known and unknown values of the global nodal forces and global nodal displacements. Table 32 displays the partitioned global structure stiffness equation matrix.

$$[F] = [K_S][U] \qquad \text{(Equation 2)}$$

$$\begin{bmatrix} F_P \\ F_Q \end{bmatrix} = \begin{bmatrix} K_{PP} & K_{PQ} \\ K_{QP} & K_{QQ} \end{bmatrix} \begin{bmatrix} U_P \\ U_Q \end{bmatrix} \qquad \text{(Equation 3)}$$

$[F_P]$ =	$[K_{PP}]$	$[K_{PQ}]$	$[U_P]$
$[F_Q]$ =	$[K_{QP}]$	$[K_{QQ}]$	$[U_Q]$

-2	226,406.38	0.00	-66,312.89	66,312.89	-66,312.89	-66,312.89	0.00	0.00	0.00	0.00	-93,780.59	0.00	U_1
0	0.00	226,406.38	66,312.89	-66,312.89	-66,312.89	-66,312.89	0.00	-93,780.59	0.00	0.00	0.00	0.00	U_2
-2	-66,312.89	66,312.89	198,938.68	-66,312.89	0.00	0.00	-66,312.89	-66,312.89	-24,272.20	90,585.10	0.00	0.00	U_3
0	66,312.89	-66,312.89	-66,312.89	198,938.68	0.00	0.00	-66,312.89	-66,312.89	24,272.20	-90,585.10	0.00	0.00	U_4
0	-66,312.89	-66,312.89	0.00	0.00	198,938.68	-66,312.89	-66,312.89	66,312.89	0.00	0.00	-66,312.89	66,312.89	U_5
-5	-66,312.89	-66,312.89	0.00	0.00	-66,312.89	198,938.68	66,312.89	-66,312.89	0.00	0.00	66,312.89	-66,312.89	U_6
0	0.00	0.00	-66,312.89	-66,312.89	-66,312.89	66,312.89	226,406.38	0.00	-81,216.38	46,890.30	0.00	0.00	U_7
-5	0.00	-93,780.59	-66,312.89	-66,312.89	66,312.89	-66,312.89	0.00	226,406.38	0.00	0.00	0.00	0.00	U_8
0	0.00	0.00	-24,272.20	24,272.20	0.00	0.00	-81,216.38	0.00	79,219.69	-73,764.63	0.00	0.00	U_9
F_{10}	0.00	0.00	90,585.10	-90,585.10	0.00	0.00	46,890.30	0.00	-73,764.63	147,186.69	0.00	0.00	0
F_{11}	-93,780.59	0.00	0.00	0.00	-66,312.89	66,312.89	0.00	0.00	0.00	0.00	160,093.49	-66,312.89	0
F_{12}	0.00	0.00	0.00	0.00	66,312.89	-66,312.89	0.00	0.00	0.00	0.00	-66,312.89	66,312.89	0

Table 32. Global structure stiffness equation matrix

3.6 Computation of Global Nodal Displacements

Using the global structure stiffness equation in Table 32, the unknown nodal displacements may be computed using Equation 4. Table 33 displays the global nodal displacement equation matrix and Table 34 displays the computed values of the global nodal displacements.

$$[U_P] = [K_{PP}]^{-1}[F_P] \qquad \text{(Equation 4)}$$

$[U_P]$ =	$[K_{pp}^{-1}]$									$[F_P]$
U_1 =	0.00001066	0.00000750	0.00000908	0.00000591	0.00000908	0.00000908	0.00000750	0.00000750	0.00000865	-2.00
U_2 =	0.00000750	0.00002471	0.00001376	0.00002038	0.00001386	0.00001834	0.00001259	0.00002155	0.00001088	0.00
U_3 =	0.00000908	0.00001376	0.00003599	0.00002782	0.00001407	0.00000877	0.00003344	0.00002284	0.00003678	-2.00
U_4 =	0.00000591	0.00002038	0.00002782	0.00003393	0.00001355	0.00001273	0.00002793	0.00002629	0.00002676	0.00
U_5 =	0.00000908	0.00001386	0.00001407	0.00001355	0.00001789	0.00001259	0.00001534	0.00001228	0.00001589	0.00
U_6 =	0.00000908	0.00001834	0.00000877	0.00001273	0.00001259	0.00002237	0.00000474	0.00001676	0.00000365	-5.00
U_7 =	0.00000750	0.00001259	0.00003344	0.00002793	0.00001534	0.00000474	0.00004128	0.00002008	0.00004401	0.00
U_8 =	0.00000750	0.00002155	0.00002284	0.00002629	0.00001228	0.00001676	0.00002008	0.00002904	0.00001953	-5.00
U_9 =	0.00000865	0.00001088	0.00003678	0.00002676	0.00001589	0.00000365	0.00004401	0.00001953	0.00006081	0.00

Table 33. Global nodal displacement equation matrix

$[U_P]$					
U_1 =	-0.0001224	m	**-0.1224**	mm	
U_2 =	-0.0002420	m	**-0.2420**	mm	
U_3 =	-0.0002482	m	**-0.2482**	mm	
U_4 =	-0.0002626	m	**-0.2626**	mm	
U_5 =	-0.0001707	m	**-0.1707**	mm	
U_6 =	-0.0002314	m	**-0.2314**	mm	
U_7 =	-0.0002060	m	**-0.2060**	mm	
U_8 =	-0.0002897	m	**-0.2897**	mm	
U_9 =	-0.0002067	m	**-0.2067**	mm	(along x''-axis)
	-0.0001790	m	**-0.1790**	mm	(along x-axis)
	-0.0001034	m	**-0.1034**	mm	(along y-axis)

Table 34. Global nodal displacements

3.7 Computation of Reaction Forces at Supports

Using the global structure stiffness equation in Table 32 and the computed global nodal displacements in Table 34, the reaction forces at supports may be computed using Equation 5. Table 35 displays the global nodal forces equation matrix and table 36 displays the computed values of the reaction forces at supports.

$$[F_Q] = [K_{QP}][U_P] \qquad \text{(Equation 5)}$$

$[F_Q]$ =	$[K_{QP}]$									$[U_P]$
F_{10} =	0.00	0.00	90,585.10	-90,585.10	0.00	0.00	46,890.30	0.00	-73,764.63	-0.0001224
F_{11} =	-93,780.59	0.00	0.00	0.00	-66,312.89	66,312.89	0.00	0.00	0.00	-0.0002420
F_{12} =	0.00	0.00	0.00	0.00	66,312.89	-66,312.89	0.00	0.00	0.00	-0.0002482
										-0.0002626
										-0.0001707
										-0.0002314
										-0.0002060
										-0.0002897
										-0.0002067

Table 35. Global nodal forces equation matrix

$[F_Q]$		
F_{10} =	6.899	kN
F_{11} =	7.450	kN
F_{12} =	4.025	kN

Table 36. Reaction forces at supports

3.8 Computation of Local Member Forces

With the computed global nodal displacements, a transformation from global to local nodal displacement is necessary to compute the local axial forces of each member. Using the local member forces formula in equation 6, the axial member forces were computed. Tables 37 to 45 display the local member forces equation matrices and the results.

$$[P_e] = [k][\beta][U] \qquad \text{(Equation 6)}$$

$P_{(1)}$ =	k_1				β_1				U	$P_{(1)}$		
$P_{11\,(1)}$ =	93780.59333	0	-93780.59	0	1	0	0	0	0.0000000	$P_{11\,(1)}$ =	11.47	kN
$P_{12\,(1)}$ =	0	0	0	0	0	1	0	0	0.0000000	$P_{12\,(1)}$ =	0.00	kN
$P_{1\,(1)}$ =	-93780.59333	0	93780.59	0	0	0	1	0	-0.0001224	$P_{1\,(1)}$ =	-11.47	kN
$P_{2\,(1)}$ =	0	0	0	0	0	0	0	1	-0.0002420	$P_{2\,(1)}$ =	0.00	kN

Table 37. Local member force of Member 1

$P_{(2)}$ =	k_2				β_2				U	$P_{(2)}$		
$P_{1\,(2)}$ =	132625.79	0	-132625.79	0	0.707106781	-0.707106781	0	0	-0.0001224	$P_{1\,(2)}$ =	9.86	kN
$P_{2\,(2)}$ =	0	0	0	0	0.707106781	0.707106781	0	0	-0.0002420	$P_{2\,(2)}$ =	0.00	kN
$P_{3\,(2)}$ =	-132625.79	0	132625.79	0	0	0	0.707106781	-0.707106781	-0.0002482	$P_{3\,(2)}$ =	-9.86	kN
$P_{4\,(2)}$ =	0	0	0	0	0	0	0.707106781	0.707106781	-0.0002626	$P_{4\,(2)}$ =	0.00	kN

Table 38. Local member force of member 2

$P_{(3)}$ =	k_3				β_3				U	$P_{(3)}$		
$P_{3\,(3)}$ =	132625.79	0	-132625.79	0	0.707106781	-0.707106781	0	0	-0.0002482	$P_{3\,(3)}$ =	**8.45**	**kN**
$P_{4\,(3)}$ =	0	0	0	0	0.707106781	0.707106781	0	0	-0.0002626	$P_{4\,(3)}$ =	**0.00**	**kN**
$P_{9\,(3)}$ =	-132625.79	0	132625.79	0	0	0	0.258819045	-0.965925826	-0.0002067	$P_{9\,(3)}$ =	**-8.45**	**kN**
$P_{10\,(3)}$ =	0	0	0	0	0	0	0.965925826	0.258819045	0.0000000	$P_{10\,(3)}$ =	**0.00**	**kN**

Table 39. Local member force of member 3

$P_{(4)}$ =	k_4				β_4				U	$P_{(4)}$		
$P_{7\,(4)}$ =	93780.59	0	-93780.59	0	1	0	0	0	-0.0002060	$P_{7\,(4)}$ =	**-2.53**	**kN**
$P_{8\,(4)}$ =	0	0	0	0	0	1	0	0	-0.0002897	$P_{8\,(4)}$ =	**0.00**	**kN**
$P_{9\,(4)}$ =	-93780.59	0	93780.59	0	0	0	0.866025404	-0.5	-0.0002067	$P_{9\,(4)}$ =	**2.53**	**kN**
$P_{10\,(4)}$ =	0	0	0	0	0	0	0.5	0.866025404	0	$P_{10\,(4)}$ =	**0.00**	**kN**

Table 40. Local member force of member 4

$P_{(5)}$ =	k_5				β_5				U	$P_{(5)}$		
$P_{5\,(5)}$ =	132625.79	0	-132625.79	0	0.707106781	-0.707106781	0	0	-0.0001707	$P_{5\,(5)}$ =	**-2.16**	**kN**
$P_{6\,(5)}$ =	0	0	0	0	0.707106781	0.707106781	0	0	-0.0002314	$P_{6\,(5)}$ =	**0.00**	**kN**
$P_{7\,(5)}$ =	-132625.79	0	132625.79	0	0	0	0.707106781	-0.707106781	-0.0002060	$P_{7\,(5)}$ =	**2.16**	**kN**
$P_{8\,(5)}$ =	0	0	0	0	0	0	0.707106781	0.707106781	-0.0002897	$P_{8\,(5)}$ =	**0.00**	**kN**

Table 41. Local member force of member 5

$P_{(6)}$ =	k_6				β_6				U	$P_{(6)}$		
$P_{11\,(6)}$ =	132625.79	0	-132625.79	0	0.707106781	-0.707106781	0	0	0	$P_{11\,(6)}$ =	**-5.69**	**kN**
$P_{12\,(6)}$ =	0	0	0	0	0.707106781	0.707106781	0	0	0	$P_{12\,(6)}$ =	**0.00**	**kN**
$P_{5\,(6)}$ =	-132625.79	0	132625.79	0	0	0	0.707106781	-0.707106781	-0.0001707	$P_{5\,(6)}$ =	**5.69**	**kN**
$P_{6\,(6)}$ =	0	0	0	0	0	0	0.707106781	0.707106781	-0.0002314	$P_{6\,(6)}$ =	**0.00**	**kN**

Table 42. Local member force of member 6

$P_{(7)}$ =	k_7				β_7				U	$P_{(7)}$		
$P_{5\,(7)}$ =	132625.79	0	-132625.79	0	0.707106781	0.707106781	0	0	-0.0001707	$P_{5\,(7)}$ =	**-3.54**	**kN**
$P_{6\,(7)}$ =	0	0	0	0	-0.707106781	0.707106781	0	0	-0.0002314	$P_{6\,(7)}$ =	**0.00**	**kN**
$P_{1\,(7)}$ =	-132625.79	0	132625.79	0	0	0	0.707106781	0.707106781	-0.0001224	$P_{1\,(7)}$ =	**3.54**	**kN**
$P_{2\,(7)}$ =	0	0	0	0	0	0	-0.707106781	0.707106781	-0.0002420	$P_{2\,(7)}$ =	**0.00**	**kN**

Table 43. Local member force of member 7

$P_{(8)}$ =	k_8				β_8				U	$P_{(8)}$		
$P_{7\,(8)}$ =	93780.59	0	-93780.59	0	6.12574E-17	1	0	0	-0.0002060	$P_{7\,(8)}$ =	**-4.47**	**kN**
$P_{8\,(8)}$ =	0	0	0	0	-1	6.12574E-17	0	0	-0.0002897	$P_{8\,(8)}$ =	**0.00**	**kN**
$P_{1\,(8)}$ =	-93780.59	0	93780.59	0	0	0	6.12574E-17	1	-0.0001224	$P_{1\,(8)}$ =	**4.47**	**kN**
$P_{2\,(8)}$ =	0	0	0	0	0	0	-1	6.12574E-17	-0.0002420	$P_{2\,(8)}$ =	**0.00**	**kN**

Table 44. Local member force of member 8

$P_{(9)}$ =	k_9				β_9				U	$P_{(9)}$		
$P_{7\,(9)}$ =	132625.79	0	-132625.79	0	0.707106781	0.707106781	0	0	-0.0002060	$P_{7\,(9)}$ =	**1.41**	**kN**
$P_{8\,(9)}$ =	0	0	0	0	-0.707106781	0.707106781	0	0	-0.0002897	$P_{8\,(9)}$ =	**0.00**	**kN**
$P_{3\,(9)}$ =	-132625.79	0	132625.79	0	0	0	0.707106781	0.707106781	-0.0002482	$P_{3\,(9)}$ =	**-1.41**	**kN**
$P_{4\,(9)}$ =	0	0	0	0	0	0	-0.707106781	0.707106781	-0.0002626	$P_{4\,(9)}$ =	**0.00**	**kN**

Table 45. Local member force of member 9

4.0 Computer-Aided Analysis

4.1 Modeling in GRASP Software

In this section, an equivalent model was presented using GRASP software to compute the displacements and forces acting on the structure. In Figure 4, an additional truss member, labeled as D- 11, was added perpendicular to the y-axis of the inclined roller support. To get accurate results, the area of the additional member was increased to a large extent to increase the stiffness of the inclined roller support equivalent representation.

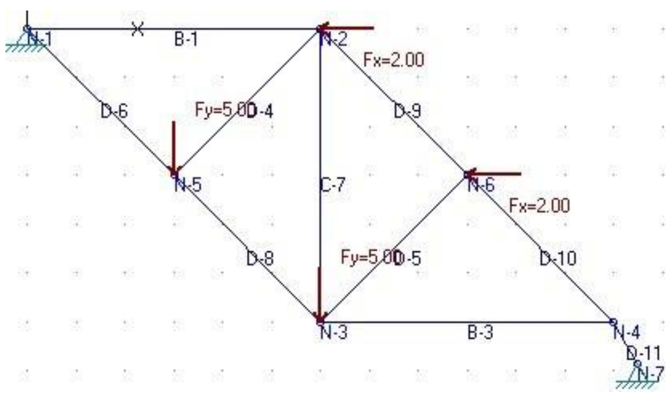

Figure 4. GRASP equivalent model

4.2 Results of the Analysis

In Figure 5 and Table 46, the resulting reactions at the supports based from the GRASP software analysis are displayed.

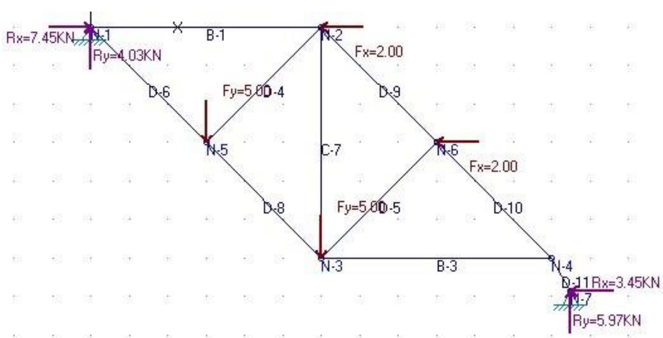

Figure 5. GRASP reaction at supports

Nodal Reactions

Live Load

Node	Fx [KN]	Fy [KN]
N-1	7.4495	4.0252
N-7	-3.4495	5.9748

Table 46. Tabulated GRASP reaction at supports

In Figure 6 and Table 47, the resulting nodal displacements based from the GRASP software analysis are displayed.

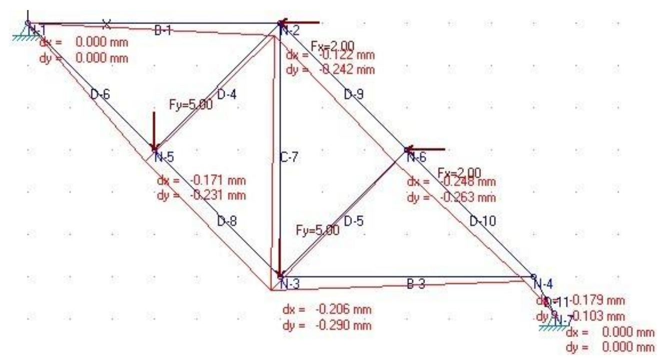

Figure 6. GRASP nodal displacements

Nodal Displacements

Live Load

Node	dx [mm]	dy [mm]
N-1	0.000000	0.000000
N-2	-0.122358	-0.241990
N-3	-0.206004	-0.289705
N-4	-0.179077	-0.103430
N-5	-0.170673	-0.231374
N-6	-0.248168	-0.262621
N-7	0.000000	0.000000

Table 47. Tabulated GRASP nodal displacements

In Figure 7 and Table 48, the resulting local member forces based from the GRASP software analysis are displayed.

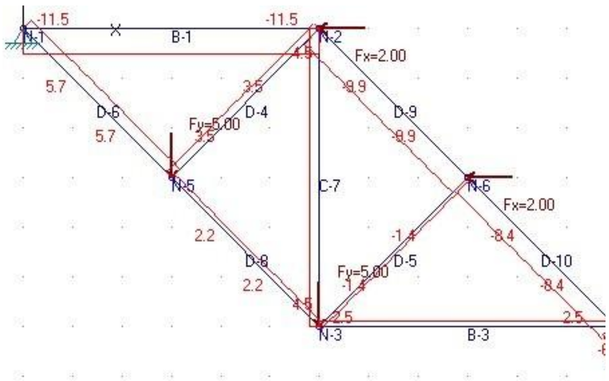

Figure 7. GRASP member forces

Member Forces

Live Load

Member	Fx.i [KN]	Fx.j [KN]
B-1	-11.4748	-11.4748
B-3	2.5252	2.5252
D-4	3.5355	3.5355
D-5	-1.4142	-1.4142
D-6	5.6925	5.6925
C-7	4.4748	4.4748
D-8	2.1570	2.1570
D-9	-9.8638	-9.8638
D-10	-8.4496	-8.4496
D-11	-6.8991	-6.8991

Table 48. Tabulated GRASP member forces

5.0 Case Study

5.1 Objectives

i. To analyze the behavior of a Plane Pratt Truss Deck Bridge supported by a hinge at one end and a roller support inclined at varying angles at the opposite end

ii. To verify if inclined supports are detrimental to a structure

5.2 Scope of the Study

i. The study was done in accordance with the given parameters and loading conditions to simplify the analysis.
ii. The truss was loaded under concentrated live loads only at the joints of the structure
iii. Self-weight and other supplemental loads were neglected
iv. An equivalent model was created using the Graphical Rapid Analysis of Structures Program (GRASP) to evaluate the static behavior of the truss.

5.3 Methodology

The analysis of the Pratt Truss Deck Bridge was executed using a two-dimensional linear elastic-truss model with the aid of the Graphical Rapid Analysis of Structures Program (GRASP). The study focused on four models having roller supports with the following angles of inclination:

i. Model A - 0 degrees inclination
ii. Model B - 30 degrees inclination
iii. Model C - 45 degrees inclination
iv. Model D - 60 degrees inclination

Modelling the Truss Bridge

For the three models with an inclined roller support, the roller support was represented by a truss element which was oriented perpendicular to its restrained axis since the GRASP software is only limited to supports that are oriented normally to the horizontal surface. Figures 8 to 11 show the different models with their corresponding roller support inclined at a certain degree. Figures 12 to 15 will display the equivalent GRASP representations of the four models, together with the reactions obtained.

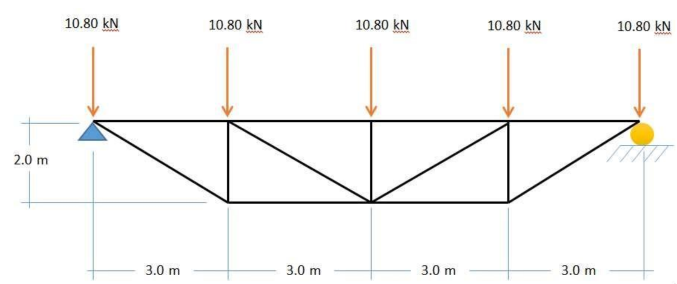

Figure 8. Model A – zero degrees inclination

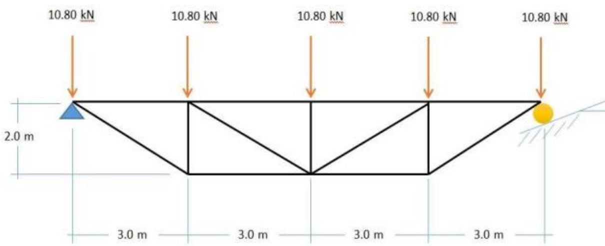

Figure 9. Model B – 30 degrees inclination

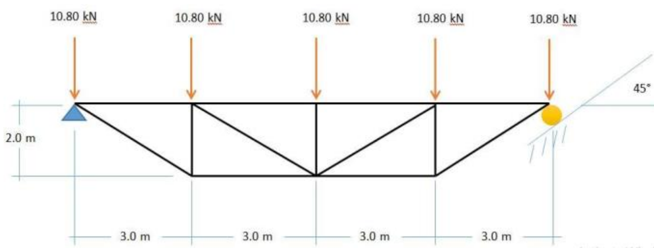

Figure 10. Model C – 45 degrees inclination

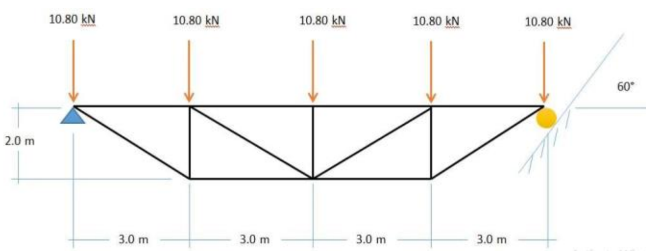

Figure 11. Model D – 60 degrees inclination

Design Parameters

The truss elements were designed using W8x31 steel members having a modulus of elasticity of 200GPa, a cross-sectional area of 5,890mm^2, and a moment of inertia of 46 x 10^6 mm^4.

Loadings

For this case study, live loads acting at the joints with a magnitude of 10.8 kN were considered. Self-weight and other supplemental loads were neglected.

5.4 **Results and Discussions**

Reaction at Supports

In Figure 12, the reactions at the supports at node N-1 and node N-5 were presented. As observed, the reactions at the supports were limited to vertical reactions since all the supports are normal to the horizontal surface, and no horizontal forces acted on the truss model.

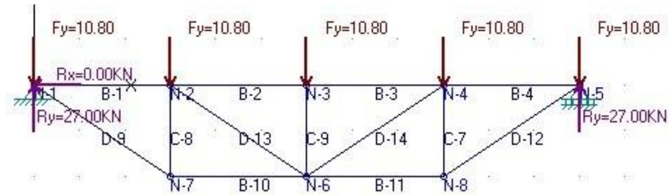

Figure 12. Reaction at supports of Model A

On the other hand, figures 13, 14, and 15 with the inclined roller support display the reactions of the hinge supports at node N-1 and node N-9. As can be observed in the three figures, there is an additional truss member D-15 representing the equivalent roller support connected to node N-5. D-15 is varyingly inclined depending on the angle of inclination of the roller support. From the reactions obtained, we notice that as the angle of inclination of the roller support increases, the horizontal reactions at the hinge supports also increase. The resultant values obtained from the reactions at node N-9 will result in the vertical reaction of the inclined roller support along its independent y-axis. From here, we can already deduce that as we increase the angle of inclination of the roller support, it becomes more stressed based on the resultant value of the reactions at node N-9 of each model.

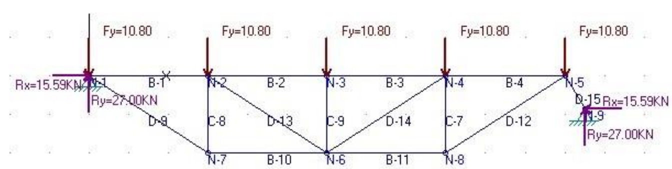

Figure 13. Reaction at supports of Model B

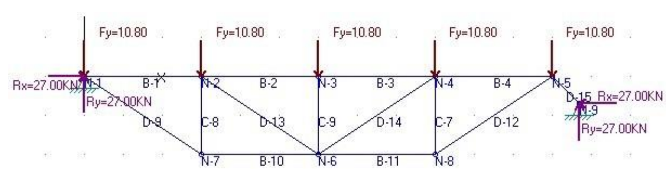

Figure 14. Reaction at supports of Model C

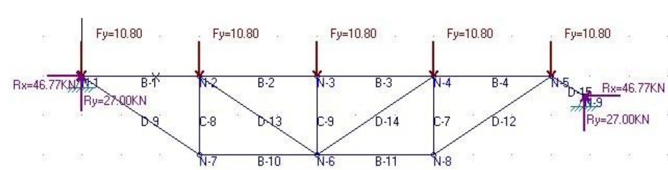

Figure 15. Reaction at supports of Model D

Local Member Forces

In Figures 16 to 19, the local member axial forces for each model are displayed. Tables 49 to 52 will display the tabulated axial forces of the members of each model.

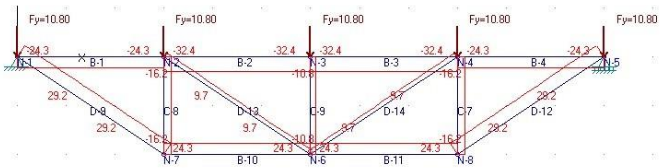

Figure 16. Axial member forces of Model A

Member Forces

Live Load

Member	Fx.i [KN]	Fx.j [KN]
B-1	-24.3000	-24.3000
B-2	-32.4000	-32.4000
B-3	-32.4000	-32.4000
B-4	-24.3000	-24.3000
C-7	-16.2000	-16.2000
C-8	-16.2000	-16.2000
D-9	29.2050	29.2050
C-9	-10.8000	-10.8000
B-10	24.3000	24.3000
B-11	24.3000	24.3000
D-12	29.2050	29.2050
D-13	9.7350	9.7350
D-14	9.7350	9.7350

Table 49. Tabulated axial member forces of Model A

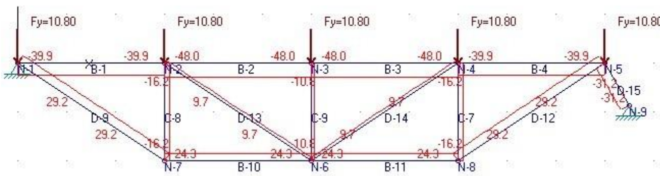

Figure 17. Axial member forces of Model B

Member Forces

Live Load

Member	Fx.i [KN]	Fx.j [KN]
B-1	-39.8885	-39.8885
B-2	-47.9885	-47.9885
B-3	-47.9885	-47.9885
B-4	-39.8885	-39.8885
C-7	-16.2000	-16.2000
C-8	-16.2000	-16.2000
D-9	29.2050	29.2050
C-9	-10.8000	-10.8000
B-10	24.3000	24.3000
B-11	24.3000	24.3000
D-12	29.2050	29.2050
D-13	9.7350	9.7350
D-14	9.7350	9.7350
D-15	-31.1769	-31.1769

Table 50. Tabulated axial member forces of Model B

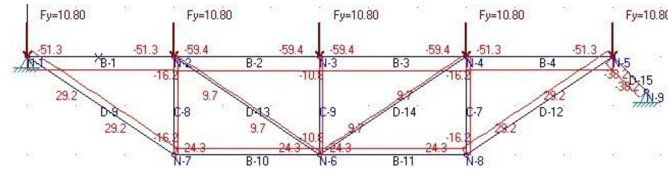

Figure 18. Axial member forces of Model C

Member Forces

Live Load

Member	Fx.i [KN]	Fx.j [KN]
B-1	-51.3000	-51.3000
B-2	-59.4000	-59.4000
B-3	-59.4000	-59.4000
B-4	-51.3000	-51.3000
C-7	-16.2000	-16.2000
C-8	-16.2000	-16.2000
D-9	29.2050	29.2050
C-9	-10.8000	-10.8000
B-10	24.3000	24.3000
B-11	24.3000	24.3000
D-12	29.2050	29.2050
D-13	9.7350	9.7350
D-14	9.7350	9.7350
D-15	-38.1838	-38.1838

Table 51. Tabulated axial member forces of Model C

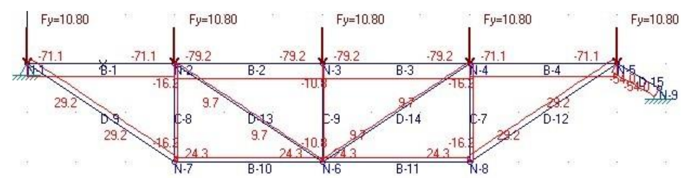

Figure 19. Axial member forces of Model D

Member Forces

Live Load

Member	Fx.i [KN]	Fx.j [KN]
B-1	-71.0654	-71.0654
B-2	-79.1654	-79.1654
B-3	-79.1654	-79.1654
B-4	-71.0654	-71.0654
C-7	-16.2000	-16.2000
C-8	-16.2000	-16.2000
D-9	29.2050	29.2050
C-9	-10.8000	-10.8000
B-10	24.3000	24.3000
B-11	24.3000	24.3000
D-12	29.2050	29.2050
D-13	9.7350	9.7350
D-14	9.7350	9.7350
D-15	-54.0000	-54.0000

Table 52. Tabulated axial member forces of Model D

Based on the results, it can be observed that the elements on top of the truss structure exhibited large axial forces as the inclination of the roller support increased. This is critical because a larger section or stiffer design is required for these members.

Nodal Displacements

In figures 20 to 23, the nodal displacements for each model are displayed. Tables 53 to 56 display the tabulated nodal displacements of each model.

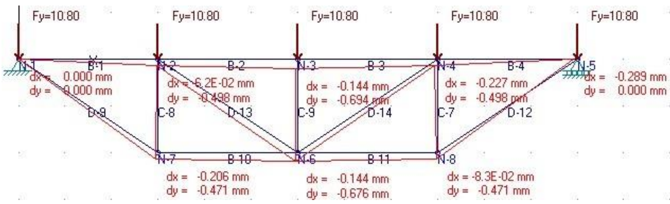

Figure 20. Nodal displacements of Model A

Nodal Displacements

Live Load

Node	dx [mm]	dy [mm]
N-1	0.000000	0.000000
N-2	-0.061885	-0.498075
N-3	-0.144397	-0.693896
N-4	-0.226910	-0.498075
N-5	-0.288795	0.000000
N-6	-0.144397	-0.675560
N-7	-0.206282	-0.470571
N-8	-0.082513	-0.470571

Table 53. Tabulated nodal displacements of Model A

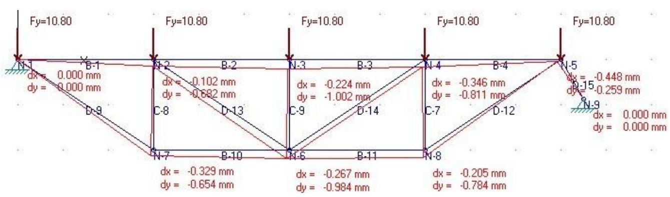

Figure 21. Nodal displacements of Model B

Nodal Displacements

Live Load

Node	dx [mm]	dy [mm]
N-1	0.000000	0.000000
N-2	-0.101584	-0.681821
N-3	-0.223795	-1.001840
N-4	-0.346007	-0.811119
N-5	-0.447590	-0.258596
N-6	-0.266895	-0.983504
N-7	-0.328779	-0.654317
N-8	-0.205010	-0.783615
N-9	0.000000	0.000000

Table 54. Tabulated nodal displacements of Model B

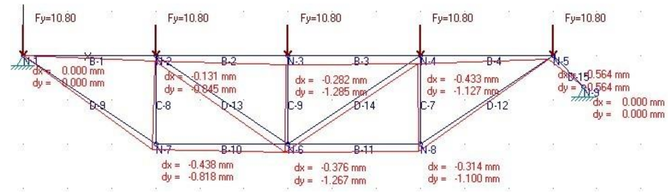

Figure 22. Nodal displacements of Model C

Nodal Displacements

Live Load

Node	dx [mm]	dy [mm]
N-1	0.000000	0.000000
N-2	-0.130645	-0.845383
N-3	-0.281918	-1.285372
N-4	-0.433192	-1.127437
N-5	-0.563837	-0.564107
N-6	-0.375936	-1.267036
N-7	-0.437821	-0.817879
N-8	-0.314052	-1.099932
N-9	0.000000	0.000000

Table 55. Tabulated nodal displacements of Model C

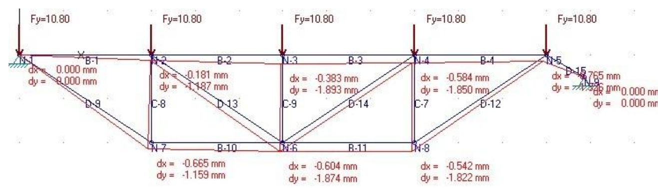

Figure 23. Nodal displacements of Model D

Nodal Displacements

Live Load

Node	dx [mm]	dy [mm]
N-1	0.000000	0.000000
N-2	-0.180981	-1.186834
N-3	-0.382591	-1.892769
N-4	-0.584201	-1.849771
N-5	-0.765182	-1.325874
N-6	-0.603570	-1.874433
N-7	-0.665455	-1.159330
N-8	-0.541686	-1.822267
N-9	0.000000	0.000000

Table 56. Tabulated nodal displacements of Model D

When designing trusses, the main consideration would be the nodal displacements since it is important to limit these variables. The displacements obtained will determine whether the structure can sustain the dead loads and live loads that are imposed on the structure. Most of the time, there are only limiting values for the displacements before the design becomes acceptable. In table 57, a comparative tabulation of the horizontal and vertical nodal displacements are presented. Also, a graphical analysis of the nodal displacements is presented in Figures 24 and 25.

Node	Displacement (mm)							
	Model A		Model B		Model C		Model D	
	x	y	x	y	x	y	x	y
1	0	0	0	0	0	0	0	0
2	-0.062	-0.498	-0.102	-0.682	-0.131	-0.845	-0.181	-1.187
3	-0.144	-0.694	-0.224	-1.002	-0.282	-1.285	-0.383	-1.893
4	-0.227	-0.498	-0.346	-0.811	-0.433	-1.127	-0.584	-1.850
5	-0.289	0	-0.448	-0.259	-0.564	-0.564	-0.765	-1.326
6	-0.206	-0.471	-0.329	-0.654	-0.438	-0.818	-0.665	-1.159
7	-0.144	-0.676	-0.267	-0.984	-0.376	-1.267	-0.604	-1.874
8	-0.083	-0.471	-0.205	-0.784	-0.314	-1.100	-0.542	-1.822

Table 57. Tabulated nodal displacements of the four models

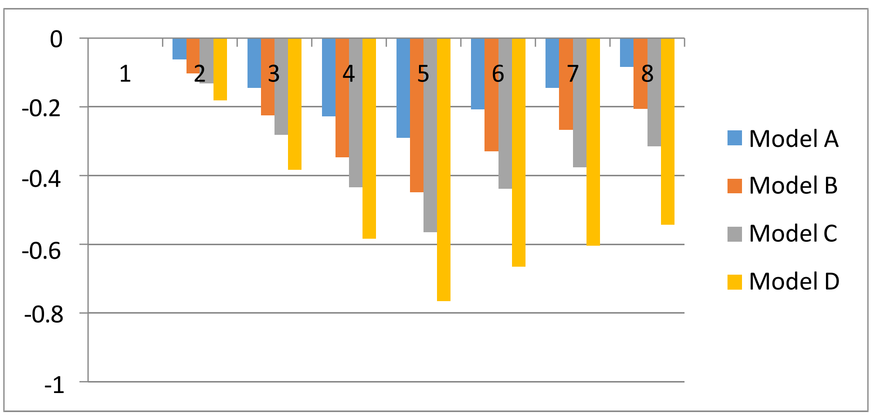

Figure 24. Horizontal displacements at each node of the four models

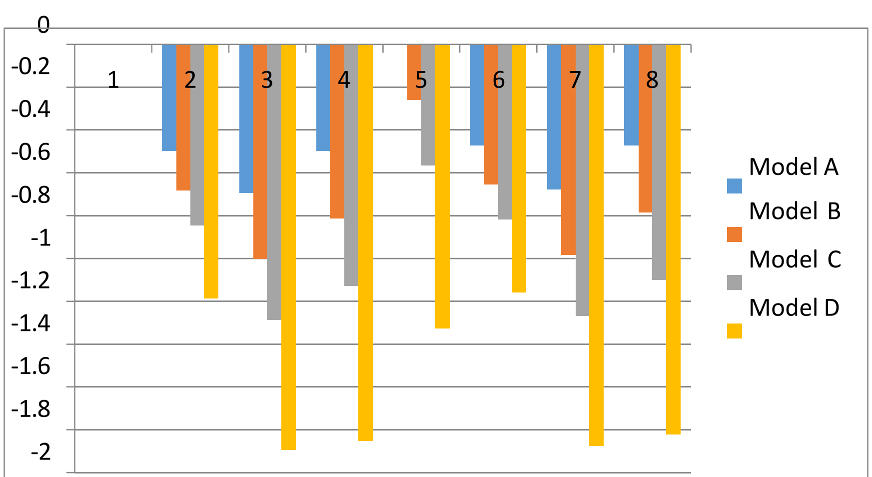

Figure 24. Vertical displacements at each node of the four models

6.0 Conclusion

On a theoretical note, the analysis of a plane truss with inclined roller supports may be done using the conventional Direct Stiffness Method (DSM) when analyzing plane trusses lying on horizontal supports. The only difference with the computation is that the transformation matrices of members that are connected to the inclined roller support will vary since the assigned global axes of the node where the inclined roller support is located are different from the global axes defined for the other nodes of the whole structure. It is important to be consistent with the defining of the global and local axes of the structure since the angle to be considered in the transformation matrices will depend on it

For the Graphical Rapid Analysis Software Program (GRASP), direct modeling of the plane truss with inclined roller supports is not possible. However, an inclined roller support may be represented by a truss element that is inclined perpendicular to the y-axis of the inclined roller support. It is important to take note that since the additional truss element is only a representation of the roller support, the area and modulus of elasticity may be increased to increase the stiffness of the member. With that, it will give more accurate results, which may be comparable to the results to be obtained using the Direct Stiffness Method.

In conclusion, when designing plane trusses, the main factors that affect the overall stability of the structure would be the properties of the truss elements such as the modulus of elasticity and the cross-sectional area. These factors determine the deflections of each node of the truss structure. However, there are other considerations that affect the stability of the truss structures. These factors include environmental impacts such as the temperature, as well as the geographical location of the structure. As observed in the case study presented

in section 5, it can be concluded that structures that are supported by an inclined roller support would be detrimental to the overall stability of the structure. It poses problems to the nodal displacements since the displacements increase as the slope of the inclined roller support increases.

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9.0 Conflict Of Interests

The authors declare no conflicts of interest

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